How the Egyptians Probably Surveyed the Pyramids

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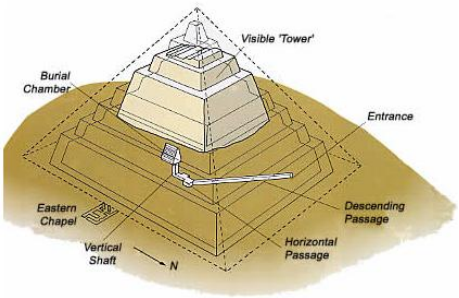
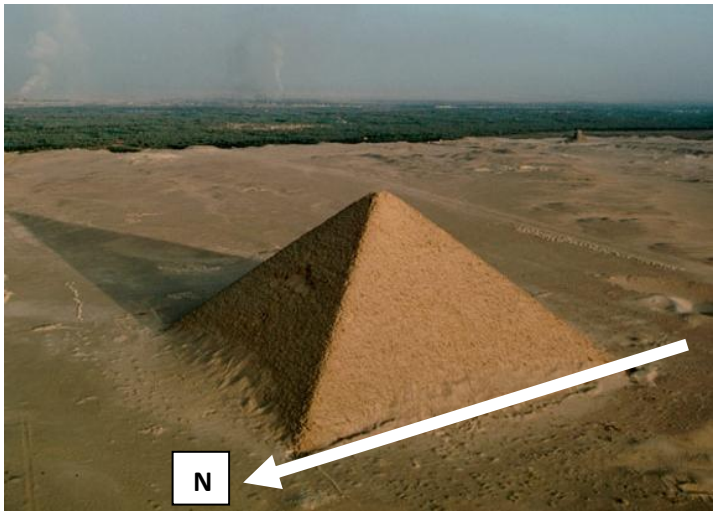
The Pyramid Design Evolves

The Pyramid of Djoser (built between 2630 and 2613 BC) was the first known monument built of cut stone. It is a stepped design standing some 62 meters high and is oriented roughly 4 degrees clockwise from north-south.[1] A north-south orientation was probably the goal, but the technique for achieving it was not yet available.



Sneferu, the first pharaoh of the 4th dynasty, modified a step-pyramid at Meidum that was apparently built by his predecessor in the old style. He added a façade of dressed stones and plaster to produce what was to become the archetype for later pyramids. Though the façade has crumbled, the core stands today, and from it we can see that this pyramid, begun probably near the end of the 3rd dynasty in about 2625 BC, is oriented in a true north-south direction.

The so-called "bent pyramid" of Sneferu (built sometime between 2612 and 2589 BC) was the first to be designed from the start as a smooth-sided pyramid. It stands 101 meters high and features a slope of 54° 27' on the lower section and 43° 22' on the upper section and is oriented in a true north-south direction. The reduced upper slope stems either from structural failure of the steep-walled lower section, or from a desire to finish the project more quickly, or with a different slope.[2] The resulting pyramid, it must be admitted, is rather clumsy and unappealing in appearance.



And finally, Sneferu constructed the first true pyramid with a single slope, the so-called "Red Pyramid" which stands 104 meters tall with a slope of 43°, identical to the upper section of the "bent pyramid". It is also oriented true north-south. So it appears that Sneferu, the first pharaoh of the 4th dynasty, perfected the construction method for true pyramids and adopted the precise north-south orientation scheme invented by his predecessor. (The shadow cast by the pyramid shows this photo was taken in the late afternoon.)

Sneferu's successor was Khufu, the builder of the Great Pyramid of Giza which, at 146.5 meters was the world's tallest structure from the time it was finished (ca. 2560 BC) until 1300 AD.[3] As can be seen in Fig. 1, the Great Pyramid of Khufu is oriented very precisely in a north-south direction. Notably, this pyramid has the aesthetically pleasing shape we now revere as the model for how a pyramid should look (51.84 ^{o} slope).

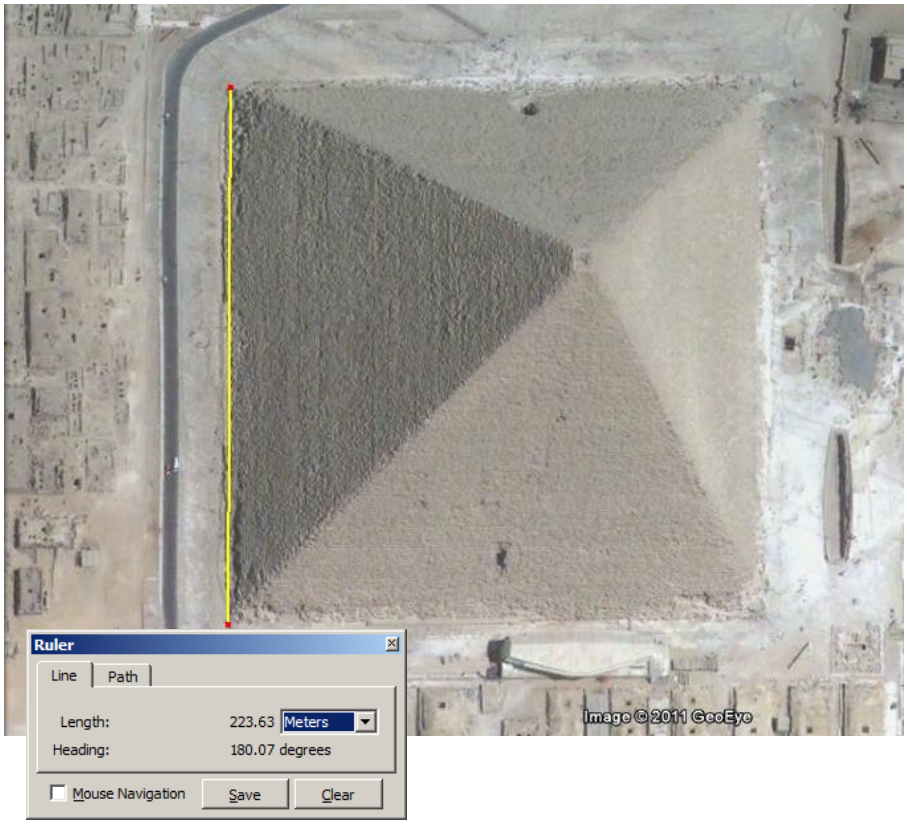
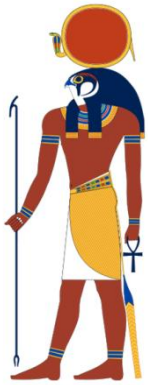


Fig. 1. The Great Pyramid of Giza, built c. 2560 BC. The Google Earth ruler shows its precise north-south orientation.

The Orientation Method

The north-south orientation of pyramids built by Sneferu and his successors is extremely accurate. How was this achieved? One possibility would have been the use of astronomical observation of the then polestar, the very faint *Thuban*. It was was less than two and a half arc-minutes away from the pole in 2787 BC, drifting to about 1 degree off-axis in 2587 BC, about the time the pyramids were built. But the pyramids are oriented to much better than 1 degree accuracy, as we saw from Figure 1. Whether the Egyptians used this pole star is unknown, but surely they placed the highest, even sacred, importance on observations of the sun.



The sun god Ra was said to travel through the heavens in his day boat then through the underworld in his night boat. Re-appearance of the sun each morning was endowed with mystical significance. Surely the very concept of east-west and, by extension, north-south came from observation of the sun. When the sun rose to its highest point at what we now call noon, it casts a shadow along the north-south line. This shadow was readily available to surveyors and would be the most practical way of determining the north-south baseline for pyramid construction. But there is a problem with this. The exact moment of maximum solar elevation cannot be determined accurately by eye, because the rate of change in elevation is slow and the shadow trajectory exhibits no clear apex. To overcome this problem, the Egyptians must have invented the techniques of what we now call *plane geometry*.

Every high school geometry student is taught how to bisect an angle. This knowledge comes to us through *Euclid*, a Greek mathematician who lived and worked in Alexandria Egypt sometime before 200 BC. His masterwork, *Elements*, collected previous geometrical knowledge and supplied logical proof as to the validity of each proposition. In Euclid's 9th Proposition, we are told how to bisect a rectilinear angle.[4] The origin of the techniques formalized by Euclid in the *Elements* lay in ancient Egypt. Ancient Egyptians made use of the *Gnomom*, a straight pole or stone monument, to cast shadows on the ground. Observing the patterns of these shadows enabled the ancient Egyptian astronomer/priest to define the four seasons precisely. And by applying the simple and intuitive method for bisecting an angle, any Egyptian surveyor or builder could lay out a very precise north-south baseline for his projects. The method described here was probably the one used to construct the pyramids of Sneferu and Khufu, though apparently not the Pyramid of Djoser, which was 4 degrees misaligned.

The method for determining a north-south baseline is diagrammed in the following two figures:

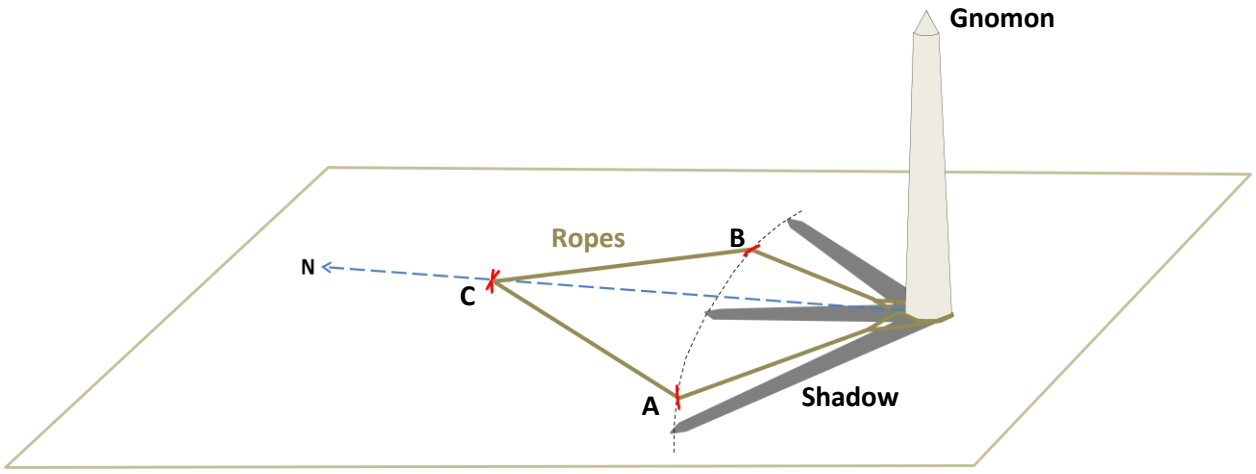


Fig. 2. The surveyor records the track of shadow tips before and after local noon, though he has no exact knowledge of time. An angle is formed by drawing circular arcs with ropes tethered to the base of the vertical gnomon, these arcs intersect the shadow trace at two points, A and B. Because the track of the shadows is symmetrical about the true north-south axis on every day of the year, these intersections form with the base of the gnomon an angle that is oriented exactly north-south. Bisecting this angle with intersecting arcs centered on the points A and B produces the point C. A rope tethered to the gnomon base is stretched over point C, forming a precise north-south baseline.

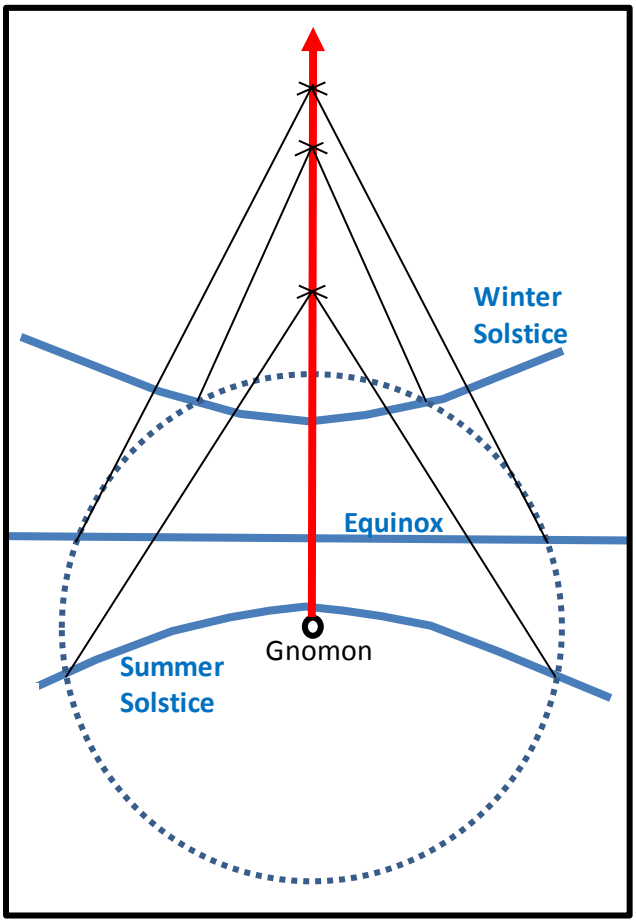


Fig. 3. The shadow traces for Giza, Egypt are shown in blue for the Summer and Winter Solstices and Vernal and Autumnal Equinoxes. An inscribed circle centered at the gnomon intersects each of the traces at points equidistant to the west and east of north. Therefore, a bisecting triangle centered on these points of intersection determines a north-south line. The technique works the same for any day of the year.

Laying out the Foundation

To a precision of less than 3 parts per thousand, the ratio of the Great Pyramid's base to its height is π/2. (Original Height=146.73m; original base=229.82m; ratio=1.566= π/2.)[5] Since the Egyptians would probably not have done high accuracy calculations with irrational numbers, and since the value of Pi (if indeed they had the concept of Pi) was probably not known to them with any accuracy, what method did they use?

Here is my theory: they first inscribed a semicircle of diameter equal to the desired height of the pyramid. They then laid a rope on the semicircle held in place by stakes or -more likely- by men. The rope of length equal to the semicircle was then straightened out along the baseline to mark the length of the base.

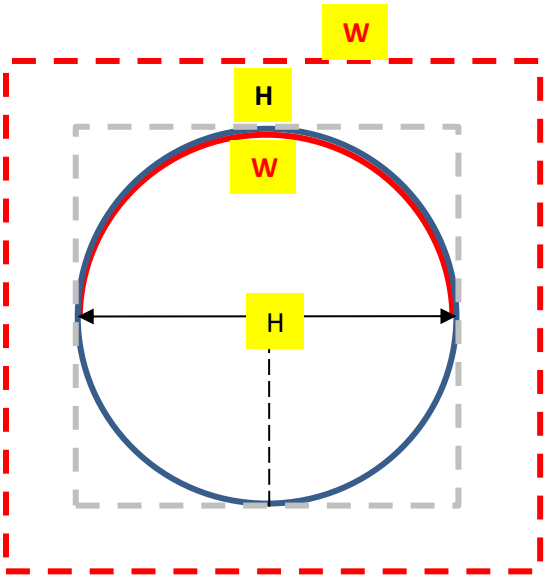


Fig. 4. The rope of length W defined the base dimension of the Great Pyramid. H was the desired height. This method was possibly chosen for reasons of mystical significance. But the height to base ratio was greater than that of the somewhat squat Red Pyramid, and by coincidence was close to the so-called "Golden Ratio" used in later Greek architecture.

With the tools described here, the builder of a pyramid could plat the north-south oriented square base quite straightforwardly, in less than a day. The following figures show the steps that could have been used.

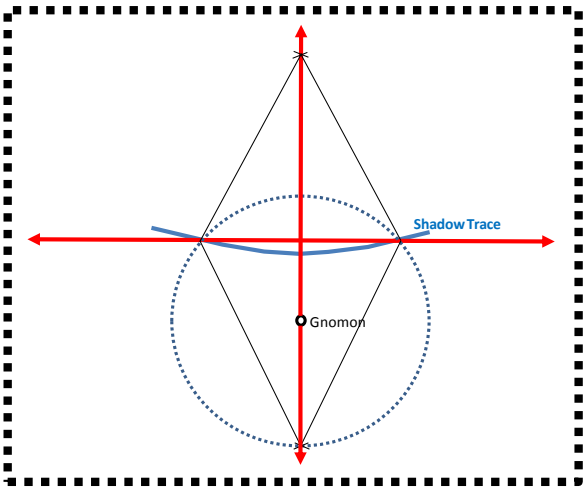


Fig. 5. The East-West and North-South baselines are determined as shown, using the solar method

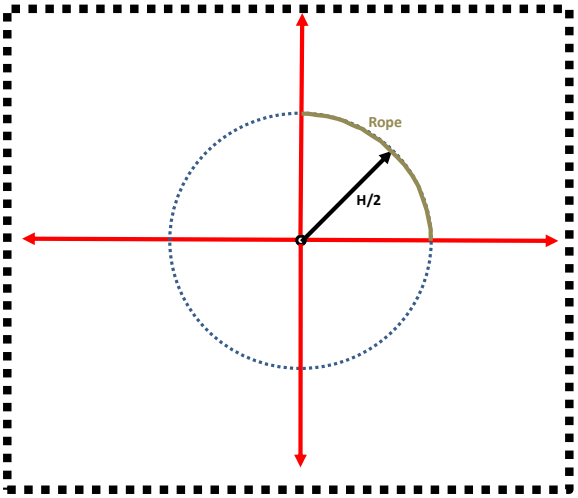


Fig. 6. A circle of diameter equal to the design height of the pyramid is inscribed with center at the intersection of the baselines. A rope is laid along one quarter of this circle to create a tool for the following steps.

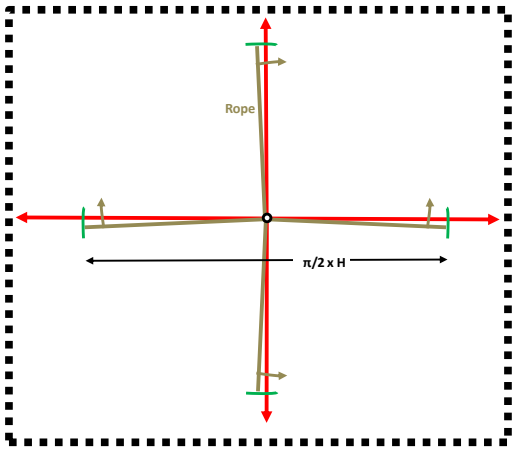


Fig. 7. The rope is used to mark the intersection of all four sides of the pyramid base with the baselines.

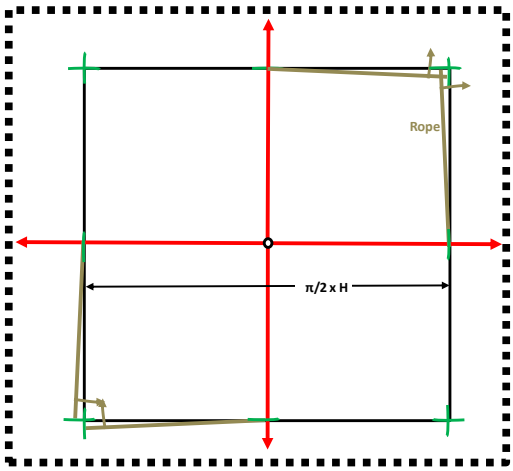


Fig. 8. The rope is used to mark the corners of the square pyramid base. The square is inscribed. They then had a north-south oriented square of side equal to Pi/2 times the design height, and the center of this square was marked for reference.

Conclusion

Whether all of these techniques were actually used by the ancient pyramid builders is of course uncertain. But the method is simple and practical and would actually be very useful even in the present day. It would be interesting to compare the results obtained with this method to those used by modern builders: GPS, laser distance measure, etc.

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[1] http://en.wikipedia.org/wiki/Pyramid_of_Djoser Orientation measured from Google Earth image.
[2] http://en.wikipedia.org/wiki/Bent_Pyramid Orientation measured from Google Earth image.
[3] http://en.wikipedia.org/wiki/Great_Pyramid_of_Giza Orientation measured from Google Earth image.
[4] *Euclid's Elements,* T.L. Heath translation, edited by D. Densmore, Green Lion Press, 2007.
[5] http://www.world-mysteries.com/mpl_2.htm cites S.G. Taseos "Back in Time 3104 B.C. to the Great Pyramid…"